Science Education International
Vol. 27, Issue 2, 2016, 253-276

Evaluating High School Students' Conceptions of the Relationship between Mathematics and Physics: Development of A Questionnaire

S. KAPUCU*, M. F. ÖÇAL, M. ŞİMŞEK

ABSTRACT: The purposes of this study were (1) to develop a questionnaire measuring high school students' conceptions of the relationship between mathematics and physics, (2) and to determine the students' conceptions of the relationship between mathematics and physics. A total of 718 high school students (343 male, 375 female) participated in this study. Espousing a phenomenographic approach, a Relationship between Mathematics and Physics Questionnaire (RMPQ) was first developed. Then explanatory factor analysis (EFA) to identify factor structure and confirmatory factor analyses (CFA) to verify the EFA results were performed on the data obtained. Data analyses showed that the RMPQ was a reliable and valid instrument to identify the students' conceptions of the relationship between mathematics and physics. Descriptive results also revealed that the majority of the students believed that mathematics and physics closely related to each other in terms of their content and daily-life relations. They also believed that mathematics was needed to understand physics.

KEY WORDS: conceptions, mathematics, physics, phenomenographic approach, questionnaire development

INTRODUCTION

It is inevitable for students who study physics to use mathematics so as to be able to solve some physics problems (Basson, 2002; Bing & Redish, 2006; Woolnough, 2000). Students need to have a deep understanding of mathematics and utilize it when necessary while engaging with physics concepts (Ataide & Greca, 2013). Even in the explanation of basic concepts in physics, students bring required components from mathematics, including symbols, structures and algebraic equations, and embed them into the problem situations by combining the mathematics and physics concepts (Ataide & Greca, 2013; Hudson & McIntire, 1977). For example, acceleration is one of the basic concepts in physics and is defined in many physics textbooks as "the rate of change of velocity" with a formula of $a = \Delta V / \Delta t$. In the teaching process, the meaning of ΔV, Δt

*Ağrı İbrahim Çeçen University, Turkey, E-mail:serkankapucu@yahoo.com

and what the ratio gives are explained in classrooms (Basson, 2002). Although students can have an understanding of mathematics and physics content separately, they may experience difficulty in applying mathematics properly and thus incorrectly interpret the physic concepts (Basson, 2002). At this point, an awareness of the importance of a close relationship between mathematics and physics becomes apparent (e.g., Heller, 1997; Kapucu, 2014a; Martinez-Torregrosa, Lopez-Gay & Gras-Marti, 2006).

This relation can be introduced in two ways. In the first place, some argue that mathematics is necessary for understanding physics (e.g., Basson, 2002; Rohrlich, 2011). This focuses the discussion on the math-dependence for understanding physics. Rohrlich (2011) looked at this situation through the eyes of the historical development of the laws of nature, which is physics. It has been argued that the laws of nature are first explained qualitatively. However, the "enormous usefulness of mathematics in the natural sciences" is emphasized by Winger (1967, cited in Rohrlich, 2011, p.3665). The state of *usefulness* of mathematics changes to a *need* component for understanding physics. This is because there is a need to generalize the laws of nature and to make them more precise. Therefore, explaining them quantitatively becomes a fundamental part of the advancement of natural sciences including physics (Rohrlich, 2011). In fact, one of the famous quotations "the laws of nature are written in the mathematical language" by Galileo also emphasizes the importance of mathematics. Apparently, physics uses mathematics as a language to introduce the laws of nature (Rohrlich, 2011).

These ideas are similar to the findings of educational studies, too. For example, Olatoye (2007) considered the mathematics as a fundamental subject for making connection among other interrelated disciplines including the natural sciences. Students' perceptions also indicated the beliefs of mathematics dependence for understanding physics. The results of a study with pre-service science and mathematics teachers about the relationship between mathematics and physics learning indicated that the highest student conceptions was that the physics was dependent on mathematics (Kapucu, 2014a). Similarly, Basson (2002) indicated that students' achievement in physics was dependent on their knowledge of mathematics and they needed mathematics to explain the physical phenomena.

Secondly, physics and mathematics had similar features, which highlighted the interrelation between them. Toluk, Uçar, Pişkin, Akkaş and Taşçı (2010) studied elementary school students' beliefs about mathematics and found that the students viewed some disciplines as similar in some aspects, particularly for the subjects, physics and mathematics. This similarity takes different forms including content (i.e., use of geometric shapes, figures, tables), use of formulas and symbols

(Nalçacı, Akarsu & Kariper, 2011), necessity of rote memorizations, excessive numbers of rules (Ornek, Robinson & Haugan, 2007), arithmetic and algebraic operations (Korsunsky, 2002). In addition, both subjects have close interaction with daily-life relations (Korsunsky, 2002). Besides, daily-life word problems are unavoidable parts of mathematics curricula and textbook (Jitendra, Griffin & Xin, 2010), but physics itself is a discipline to explain the natural world (Munier & Merle, 2009). From the point of required skills for students to solve problems, Hermann (1991) discussed that mathematics and physics share common features. For example, students need to use their critical thinking skills to formulate the situation, to interpret the data given, to reach conclusion, and to make a generalization about the presented situation (Hermann, 1991). Moreover, this idea is supported with university students' conceptions based on their physics and mathematics achievement levels. According to their conceptions, students who are successful in mathematics are also probably successful in physics (Kapucu, 2014a). Moreover, students' insufficient knowledge of mathematics can negatively influence their attitudes toward physics (Kapucu, 2014b; Semela, 2010). Students might not choose a physics course in their education (Semala, 2010) and they might even hate physics (Kapucu, 2014b) due to their low-level achievement in mathematics.

The relationship between these disciplines also influences students' learning. Although physics uses mathematics as a language to explain the natural world, its use of numbers, variables and equations differs when comparing it with mathematics application (Basson, 2002). While students combine content of mathematics and physics, and apply components of mathematics in explaining physical phenomena, the knowledge of mathematics that students hold in their mind gains importance. They no longer consider the knowledge of physics and mathematics as separate topics. That means students use the knowledge of mathematics by giving attention to its meaning in the corresponding knowledge of physics and in interpreting it (Bing & Redish, 2006). Fauconnier and Turner (2002) explain this situation with conceptual blending that is about students' cognitive processes in combining mathematics and physics. The blending process appears when there is a match between the inputs from two separate mental spaces. The process continues with constructing new blended mental space. Therefore, new relations and meanings between two mental spaces can be formed (Bing & Redish, 2006). Considering the classification of Bing and Redish (2006) about blending the knowledge of mathematics and physics, in acceleration example discussed before, the positive and negative quantities, the multiplication rules of opposite signs of the quantities, and algebraic symbols of a, ΔV, and Δt are all knowledge of mathematics. On the other hand, up and down directions, change of velocity and time, and acceleration are the knowledge of physics. When students blend the knowledge, they create a new emergent structure and give meaning to the formula (Bing & Redish, 2006). They create a relationship that the positive sign means ‘up direction’ and negative sign means ‘down direction’. In addition, they also develop understanding about what decreasing and increasing accelerations mean by blending the knowledge of mathematics and physics (Bing & Redish, 2006). Sometimes, students begin with components of mathematics and translate them into physical ideas. The inverse may also appear during studying a problem situation. For example, when students try to solve problems in physics, they need to know not only the sufficient knowledge of mathematics, but also how to utilize it. Therefore, students need to see close interrelation between mathematics and physics in succeeding in doing so (Bing & Redish, 2006).

In many cases, learning does not always appear at an expected level when studying physics that requires mathematics. Students may struggle with physics during blending, or combining it with mathematics (Clay, Fox, Grunbaum & Jumars, 2008). There are several reasons of such difficulties according to the scholars. Transferring the knowledge of mathematics into physics is not an easy task to accomplish for students (Mestre, 2001; Woolnough, 2000). It is very common that students use formulas and perform numeric computations but do not know what these procedures stand for in physics (Martinez-Torregrosa et al., 2006). After learning new knowledge in physics, students are required to analyze them among multiple contexts and relate them to previously learned knowledge by using necessary parts of knowledge of mathematics (Munier & Merle, 2009). For example, the National Council of Teachers of Mathematics (NCTM, 2000) in USA emphasizes “to apply geometric ideas and relationships to other areas of mathematics, to other disciplines, and to problems that arise from their everyday experiences” (p.169) in school mathematics standards. However, this is not always put into practice in school environments. Students often have difficulties in linking graphs, or diagrams to physic concepts or to the real world (Basson, 2002). In addition, students believe that they need higher levels of mathematics skills to do physics. This influences students’ attitudes toward learning physics negatively (Ornek et al., 2007).

In fact, the teaching of mathematics, physics, or any other discipline in science is taught separately in schools (Clay et al., 2008). Therefore, it is very possible for students in schools to have a belief that mathematics and physics can be considered as unrelated subjects. Even with the universities, it is very common in engineering or science faculties providing courses such as “mathematics for physics/engineering/science.” This initiates a discussion whether there is a need of preparing students for specific mathematics content in such faculties to learn science. Therefore, there is a presupposition that only some parts of mathematics are necessary to learn science and the rest is unrelated to it (Clay et al., 2008). However, there is no consensus on whether integration of components of mathematics into science including physics has an improving effect on students' understanding of the content taught, or a confusing effect on their learning of abstract mathematical principles or scientific phenomena (Kim & Aktan, 2014).

SIGNIFICANCE OF THE STUDY

The present literature indicates different dimensions about the strong interrelation between physics and mathematics, and between learning mathematics and physics. This relation includes mathematics dependence on physics (e.g., Ataide & Greca, 2013; Basson, 2002; Kapucu, 2014a) and the similarities between them (e.g., Korsunsky, 2002; Munier & Merle, 2009). Moreover, the difficulties of learning physics, or science in general due to mathematics related reasons, are also widely discussed before (e.g., Clay et al., 2008; Kapucu, 2014b; Martinez-Torreggosa et al., 2006; Mestre, 2001). In fact, there are studies that investigate the relationship between mathematics and physics from pre-service teachers' conceptions or beliefs (e.g., Ataide & Greca, 2013; Kapucu, 2014a). However, the studies investigating this relationship from the conceptions of physics learners, who are high school students, are limited. Although students learn some physics topics as a requirement of science course in elementary school, it is the first time for them to learn physics as a separate discipline in high school. Therefore, with the knowledge of mathematics and physics that students learn in high school, they can be more prone to analyze the relationship between them.

In many occasions, the relationship between mathematics and physics is discussed (Basson, 2002). When the literature is reviewed about the relationship between mathematics and physics, it is observed that the studies examining the different perspectives (e.g., Ataide & Greca, 2013; Kapucu, 2014b; Martinez-Torregrosa et al., 2006) mostly discuss the relationships without making any detailed classifications for such conceptions. Therefore, there appears a need to investigate this relationship in more comprehensive and holistic ways. In addition, considering the fact that the possible negative influence of insufficient knowledge of mathematics on students' attitudes toward physics (Semela, 2010; Kapucu, 2014b), there is a need for a general platform to see students' conceptions of the relationship between mathematics and physics. This study is trying to fill this gap in the literature with a questionnaire measuring high school students' conceptions of the relationship between mathematics and physics.

With this questionnaire, students' conceptions, about this issue, can be identified allowing researchers to determine the general tendency among students. This is important to fulfill effective teaching of physics in classroom environments. For example, teachers maybe more careful in teaching physics content regarding the students' conceptions. Teaching physics by being aware of such conceptions may thus increase the effectiveness of teaching methods and strategies that teachers use. In addition, the findings of this study can imply the reasons why students have difficulty in learning physics. Teachers can take precautions to eliminate students' possible obstacles in learning physics by realizing these reasons that might negatively affect students' learning of physics. This study may also help the researchers studying on the relationship between mathematics and physics, and curriculum developers. From the researchers' perspective, this study may give cues for those who study factors affecting students' success in physics, and their beliefs and attitudes related to physics and physics learning. In addition, they can be aware of students' conceptions about the relationship between mathematics and physics before conducting interdisciplinary studies. For curriculum developers, the finding of this study can show the importance of mathematics in learning physics, or in general, in learning science. In addition, this study can also show that mathematics and physics are not entirely separate from each other. It can be claimed that integrated teaching programs might be needed to make students comprehensively understand the topics in physics.

PURPOSE OF THE STUDY

The purpose of this study is twofold;

1) to develop a questionnaire measuring high school students' conceptions of the relationship between mathematics and physics,
2) to determine the students' conceptions of the relationship between mathematics and physics.

METHOD

Sample

A total of 718 high school students (343 male, 375 female) from seven different schools enrolled in a city located in Eastern region of Turkey participated in this study. According to the Turkish Education System, students are guided to study in different types of high schools according to their achievements in nationwide exams that have been carried out during students' education in elementary schools. These exams aim to measure students' cognitive skills and they include multiple choice test items. For example, more successful students in these exams, have a right to enroll in the high schools mostly including high-achieving students in these exams. Considering this fact, we try to select seven different high schools having students who are in different achievement levels. These schools are also chosen from different regions of the city (e.g., city center, suburban areas) to reach a variety of different social-economic backgrounds. Furthermore, according to Turkish education system, national curricula are implemented in all of the regions of the country. The Ministry of National Education put the rules as an authority and teachers teach accordingly. Therefore, almost all the students in Turkey probably receive a very similar education in all of the regions of the country. Thus, to a certain extent, the surveyed students in the study represent the variation in the profiles of Turkish high school students. There is also approximately equal number of students in all grades (9, 10, 11, and 12).

Development of a Questionnaire

In the development of a questionnaire, a phenomenographic approach is used. Phenomenography explores the different conceptions that learners hold in their minds as a result of their experiences during learning (Entwistle, 1997; Ornek, 2008). In other words, it investigates how individuals conceptualize and understand the different features of phenomena in the world (Ornek, 2008) and it focuses on individuals' experiences of learning (Linder & Marshall, 2003). Such an approach is also used by some researchers (Tsai, 2004; Eklund-Myrskog, 1998) to identify students' conceptions of learning. Although this study does not aim to investigate this, the data collection tools of this study aim to measure outcomes of students' learning experiences in mathematics and physics. This approach basically includes data gathering by interviewing with individuals in a semi-structured manner (Ornek, 2008). However, in this study, as well as interviewing students, an open-ended questionnaire to identify students' conceptions of the relationship between mathematics and physics is developed. In this questionnaire, students are asked to explain the relationships, similarities, and differences between mathematics and physics according to their learning experiences in these disciplines. This questionnaire is administered to 108 high school students (50 male, 58 female).

In addition, a group interview with nine students within this sample was conducted to obtain deeper information about the students' conceptions. In the interviews, students were asked the same questions as in the open-ended questionnaire. However, they were conducted in a semi-structured manner by using extra questions such as "why do you think so", or "could you explain that further". According to students' responses to both the open-ended questionnaire and interview questions, identified items were used to develop an initial version of the questionnaire measuring students' conceptions of the relationship between mathematics and physics. The questionnaire was entitled, the Relationship between Mathematics and Physics Questionnaire (RMPQ). A total of 28 items were obtained, which were anchored as a five-point Likert mode (strongly agree=5, agree=4, neither agree nor disagree=3, disagree=2, strongly disagree=1). This version of the questionnaire was delivered to 272 students, in three different schools. Item analysis and explanatory factor analysis (EFA) were performed and following this, 24 items were retained. After almost two months, this questionnaire was delivered to 338 students in another four different schools. Confirmatory factor analysis (CFA) was carried out to verify the results obtained from EFA. The framework of the research procedure was as presented in figure 1.

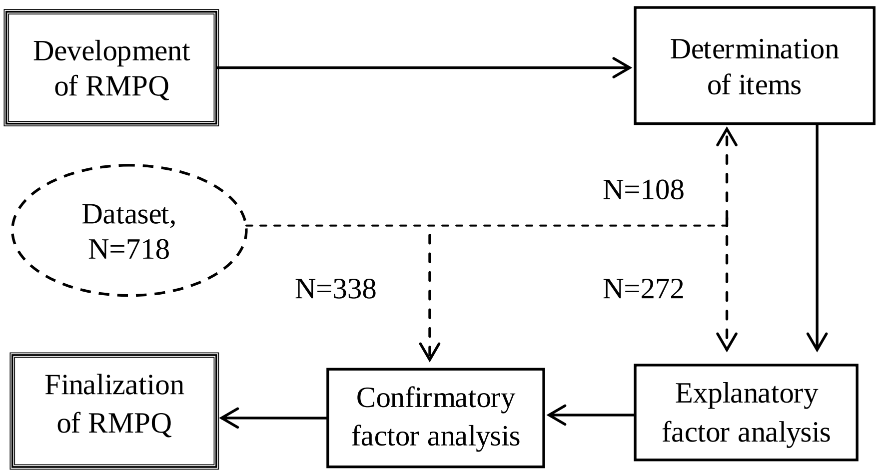

Figure 1. Framework of the research procedure

Data Analysis

Categories and codes were constructed according to student responses to the open-ended questionnaires. Bogdan and Biklen (1998) suggested that examining data considering regularities and patterns, as well as for topics that the data covered, could be helpful for achieving the codes and categories. In this regard, statements and key words, characterizing the conceptions in the open-ended questionnaires, were underlined. Audio-taped interviews were also transcribed and extra codes representing the conceptions were identified. Similar to the analysis of the open-ended questionnaire, phrases and keywords, representing the conceptions, were identified. Then, considering the similarities and differences in the statements and the key words, five categories were labeled. After the discussion on items to be included in the questionnaire, the new questionnaire format was prepared.

The questionnaire, consisting of 28 items, was tested with 272 high school students and item analysis carried out. According to Büyüköztürk (2011), the minimum value for total-item correlation indices could be suggested as 0.2 for each item. Based on this, four items were removed from the questionnaire. EFA was then performed on the remaining data. Keeping the EFA suggestions by Worthington and Whittaker (2006) in mind, items having factor loads over 0.4 were kept which resulted in a questionnaire having 24 items in 5 factors. In addition, following suggestions by Costello and Osborne (2005) and Worthington and Whittaker (2006), on the development of a questionnaire, the questionnaire obtained at the end of the EFA was administered to a different student sample consisting of 338 high school students.CFA was carried out on this data. According to Byrne (2010), fit indices needed to be evaluated to test the proposed model. For this study, chi square per degree of freedom (CMIN/df), root mean square error of approximation (RMSEA), goodness-of-Fit-Index (GFI), comparative fit index (CFI), and Tucker-Lewis index (TLI) were used. Acceptable values for GFI were taken to be above 0.90, for CFI and TFI above 0.95, for CMIN/df between 0 and 3, and for RMSEA between 0 and 0.08 (Schermelleh-Engel & Moosbrugger, 2003). For the reliability (Pallant, 2005), Cronbach alpha coefficients were calculated for each factor obtained from EFA and CFA, and test-retest reliability coefficients, using 30 students in the sample, were calculated.

RESULTS

Identification of the Items

Table 1 presents the conceptualized categories, identified items for each category and some example quotes from students' responses to the open-ended questionnaire and interview. As illustrated in Table 1, five different conceptions about the relationship between mathematics and physics are identified.

Table 1. Conceptualized Categories, Identified Example Items for Each Category, and Example Quotes from Students' Responses Representing Their Conceptions

Categories	Identified items	Example quotes
Content similarity conceptions (CSC)	Mathematics and physics are mostly based on memorization. Mathematics and physics mostly use rules or formulas.	Mathematics and physics; both include lots of rules. If you do not know them, you cannot solve the questions. Particularly, in physics, there are lots of formulas to be memorized. (IR) Mathematics and physics are abstract. I think

	Mathematics and physics include abstract concepts.	that learning abstract contents is difficult because you cannot imagine them. (IR) Both subjects include numerical calculations (OQR)
Mathematics dependence conceptions (MDC)	Physics topics are not deeply understood without knowing mathematics. Successfulness in physics substantially depends on knowing mathematics.	In the nature, there are phenomena related to physics. To understand them we use mathematics. (OQR) Physics is composed of rules and formulas. They all reach significance due to mathematics calculations. Without knowing mathematics, nobody understands these rules. (IR)
Content difference conceptions (CDC)	Physics includes more rules than mathematics to be memorized. Mathematics includes more precise results than physics.	Physics emerges with scientists' ideas and then it is shaped but mathematics is always precise. (OQR) Physics needs more memorization due to having lots of formulas and rules. In contrast, mathematics is based on logic. You do not need to memorize everything. (IR)
Daily-life similarity conceptions (DSC)	Mathematics and physics play an important role to overcome the problems in our life. Mathematics and physics help us to improve our living conditions.	Both are part of our life and we frequently use them to meet our needs. (OQR) Everything in our life is related to them. Their nonexistence can cause serious problems. Machines cannot work and that means life becomes paralyzed. (IR)
Daily-life difference conceptions (DDC)	Physics is more related to natural phenomena than mathematics. Mathematics is more used in daily life than physics.	Physics examines the rules on Earth more and benefits from mathematics to find the solutions related to them; it uses mathematics. (OQR) I think that mathematics are more related to daily-life. In the simplest way, we use numbers to call someone on the phone. (IR)

Note: IR refers to quote taken from interview response, and OQR refers to quote taken from an open-ended questionnaire response.

Item Analysis

Before executing EFA, item analysis was performed to examine corrected item-total correlations. This analysis was performed with a total of 272 cases. Table 2 reveals the results obtained from item analysis.

Table 2. Results of an Item Analysis

Item	Corrected Item-Total Correlation	Cronbach's Alpha /if Item Deleted		Corrected Item-Total Correlation	Cronbach's Alpha/ if Item Deleted
I1	0.34 / 0.35*	0.83 / 0.86**	I15	0.49 / 0.48*	0.82 / 0.85**
I2	0.56 / 0.58*	0.82 / 0.85**	I16	0.30 / 0.29*	0.83 / 0.86**
I3	0.45 / 0.45*	0.82 / 0.85**	I17	0.46 / 0.47*	0.82 / 0.85**
I4	0.22 / 0.21*	0.83 / 0.86**	I18	0.39 / 0.38*	0.82 / 0.85**
I5	0.24 / 0.26*	0.83 / 0.86**	I19	0.53 / 0.51*	0.82 / 0.85**
I6	0.39 / 0.39*	0.82 / 0.85**	I20	0.29 / 0.32*	0.83 / 0.86**
I7	0.53 / 0.54*	0.82 / 0.85**	I21	0.48 / 0.50*	0.82 / 0.85**
I8	0.58 / 0.59*	0.82 / 0.85**	I22	0.29 / 0.32*	0.83 / 0.86**
I9	0.62 / 0.64*	0.82 / 0.85**	I23	0.35 / 0.36*	0.83 / 0.85**
I10	0.24 / 0.26*	0.83 / 0.86**	I24	0.45 / 0.45*	0.82 / 0.85**
I11	0.44 / 0.46*	0.82 / 0.85**	R1	0.01	0.84
I12	0.45 / 0.47*	0.82 / 0.85**	R2	0.06	0.84
I13	0.41 / 0.42*	0.82 / 0.85**	R3	0.03	0.84
I14	0.28 / 0.31*	0.83 / 0.86**	R4	0.05	0.84

*Corrected item-total correlation coefficient after removing R1, R2, R3 and R4 in the data
**Cronbach's alpha if item deleted after removing R1, R2, R3 and R4 in the data

As Table 2 indicates corrected item-total correlation coefficients of the items are between 0.01 and 0.62. The last four items (R1, R2, R3 and R4 referring to removed items) in Table 2 are removed from RMPQ due to their low item-total correlation indices. The optimum suggested value for the correlation coefficient is 0.2 (Büyüköztürk, 2011). The remaining data show that the correlation coefficients ranged from 0.21 to 0.64.

Validity of RMPQ

EFA Results of RMPQ

EFA was carried out on 24 items in the remaining data obtained after an item analysis. The results showed that the KMO measure of sampling adequacy index was 0.847, and Bartlett's test of sphericity was statistically significant (χ^2 = 2104,026 df=276, p<0.0001). The values for the KMO index were over 0.6 and the significant Bartlett's test was a prerequisite to perform EFA (Pallant, 2005). Therefore, these values implied that EFA could be performed with this data. A principal component analysis with varimax rotation was carried out to identify the factors. The option of eigenvalues greater than one was selected. Five factors were retained as previously proposed according to findings obtained by the open-ended questionnaire and interview. The factor structure by representing the items' means, standard deviations and communalities as well as the variance of each factor was as shown in Table 3.

Table 3. Factor Structure, Items' Means, Standard Deviations and Communalities as well as Variance of Each Factor

	Factor 1	Factor 2	Factor 3	Factor 4	Factor 5			
Item	P	P	P	P	P	M	SD	h2
Factor 1: Content similarity conceptions (CSC)								
I17	**0.76**	0.05	0.10	-0.06	-0.05	3.84	1.35	0.60
I7	**0.72**	0.07	-0.06	0.09	0.22	3.97	1.20	0.47
I8	**0.71**	0.13	-0.03	0.30	0.06	3.87	1.29	0.78
I11	**0.69**	0.09	-0.09	-0.02	0.15	3.76	1.27	0.68
I9	**0.67**	0.30	0.18	0.07	0.04	3.92	1.41	0.58
I21	**0.66**	-0.03	0.09	0.19	0.11	3.89	1.13	0.56
I19	**0.60**	0.23	0.09	0.13	-0.05	3.71	1.45	0.62
I2	**0.59**	0.08	0.16	0.27	0.23	3.75	1.27	0.57
I12	**0.59**	-0.07	0.09	0.14	0.32	4.09	1.27	0.62
Factor 2: Mathematics dependence conceptions (MDC)								
I1	0.02	**0.75**	0.03	0.05	0.10	3.13	1.36	0.49
I23	0.07	**0.64**	0.07	0.11	0.05	3.34	1.20	0.51
I18	0.20	**0.61**	0.20	-0.09	-0.06	3.10	1.46	0.43
I16	0.07	**0.61**	-0.01	-0.15	0.28	3.35	1.37	0.77
I5	-0.00	**0.56**	0.17	0.14	-0.14	2.99	1.28	0.58
I15	0.26	**0.53**	0.07	0.10	0.22	3.51	1.29	0.41
Factor 3: Content difference conceptions (CDC)								
I22	0.04	0.11	**0.87**	0.07	0.01	3.27	1.33	0.49
I13	0.20	0.15	**0.84**	-0.00	0.02	3.23	1.36	0.48
I20	0.01	0.17	**0.78**	0.02	0.11	3.23	1.30	0.50
Factor 4: Daily-life similarity conceptions (DSC)								
I4	0.03	-0.02	0.01	**0.82**	0.01	3.33	1.41	0.65
I24	0.32	0.17	-0.06	**0.69**	0.06	3.53	1.27	0.44
I3	0.32	0.04	0.17	**0.67**	0.04	3.57	1.25	0.47
Factor 5: Daily-life difference conceptions (DDC)								
I10	0.18	0.01	-0.01	-0.06	**0.73**	3.46	1.21	0.58
I14	0.00	0.25	0.20	0.03	**0.61**	3.33	1.25	0.48
I6	0.30	0.05	-0.04	0.18	**0.60**	3.50	1.22	0.38
% of Variance	25.21	10.93	7.19	6.23	5.21			

Note: P=pattern coefficients, M=mean, SD=Standard Deviation, h2=communalities

According to Table 3, the minimum factor load is 0.53 and the factor loads vary between 0.53 and 0.87. When the communalities are examined, the minimum value was found to be 0.38 for item I6. The agreed literature view for commonality value was that it was over 0.50 (Worthington &

Whittaker, 2006) while Costello and Osborne (2005) argued that using the communality values over 0.40 for an item was a better choice to accept an item in the factor. In this study, I6was not eliminated from RMPQ considering an adequate factor load of it in EFA and the questionnaire's content validity. Furthermore, these five factors explained a total of 54.78% of the variance. According to Scherer, Wiebe, Luther and Adams (1988), the values for total variance explained above 40% were acceptable. Thus, the value found for the total variance explained in this study could be considered as acceptable.

CFA Results of RMPQ

CFA was carried out on the data obtained from 338 students to evaluate whether the data would justify the EFA results. The Amos program was utilized to test the proposed model. In this model, factors were depicted as latent variables and the items loaded on these factors were depicted as observed variables. The Amos output representing these variables was as presented in figure 2.

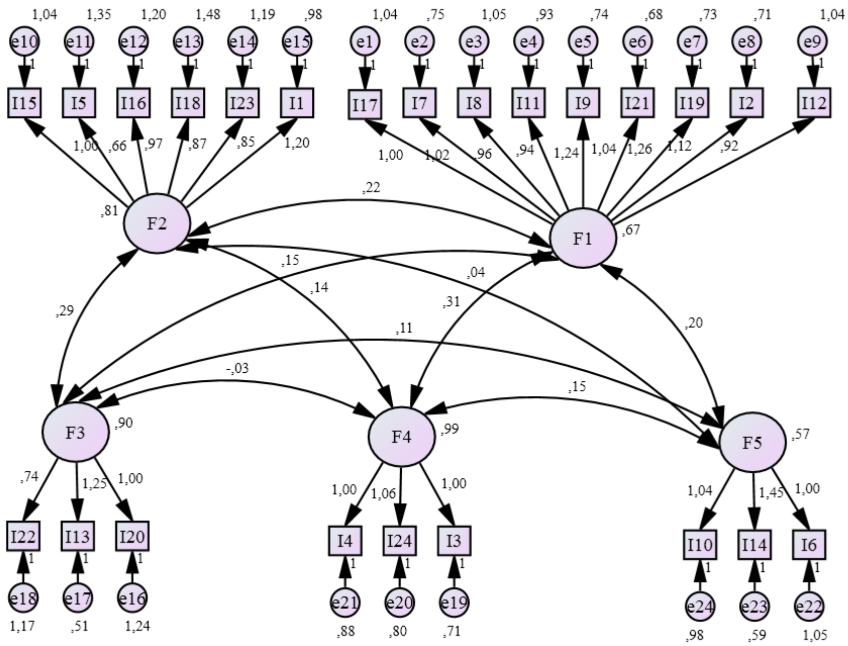

Figure 2. CFA path diagram

Standardized regression weights (factor loads) were calculated to be able to observe the relationship between latent and observed variables (Byrne, 2010). Measurement errors associated with observed variables and significance levels of each factor load was also checked. The factor loads, measurement errors, and significance levels were as depicted in Table 4.

Table 4. Factor Loads (FL), Measurement Errors (ME), and Significance Levels (p)

Factor	Item	FL	ME	p
F1: CSC	I7	0.69	0.10	<0.001
	I8	0.61	0.10	<0.001
	I11	0.63	0.10	<0.001
	I9	0.76	0.11	<0.001
	I21	0.72	0.10	<0.001
	I19	0.77	0.11	<0.001
	I2	0.74	0.10	<0.001
	I12	0.59	0.10	<0.001
	I17	0.63	-	-
F2: MDC	I1	0.74	0.12	<0.001
	I23	0.57	0.10	<0.001
	I18	0.54	0.11	<0.001
	I16	0.62	0.10	<0.001
	I5	0.46	0.10	<0.001
	I15	0.66	-	-
F3: CDC	I22	0.54	0.09	<0.001
	I13	0.86	0.15	<0.001
	I20	0.65	-	-
F4: DSC	I4	0.73	0.09	<0.001
	I24	0.76	0.09	<0.001
	I3	0.76	-	-
F5: DDC	I10	0.62	0.13	<0.001
	I14	0.82	0.17	<0.001
	I6	0.59	-	-

As shown in Table 4, all factor loads were significant and the measurement errors were smaller than 0.20. The minimum factor load value was also 0.46. Moreover, some fit indices CMIN/df, RMSEA, GFI, CFI, and TLI were examined with Amos. The fit indices, CMIN/df, RMSEA, GFI, CFI, and TLI, were found to be 1.458, 0.037, 0.921, 0,957, and 0,951, respectively. Given the fact that the fit indices GFI over 0.90, CFI and TFI over 0.95, CMIN/df between 0 and 3, and RMSEA between 0 and 0.08 were considered as acceptable for model fit (Schermelleh-Engel & Moosbrugger, 2003), it could be affirmed that the obtained indices in the study met these requirements. In this regard, these fit indices could indicate a reasonable fit and confirmed the EFA results.

Reliability of RMPQ

Internal Consistency

The Cronbach alpha reliability coefficient of each factor (factor 1, 2, 3, 4, and 5) was calculated on the data obtained for EFA and CFA to test the reliability of the RMPQ. For EFA data obtained, the reliability coefficients were found to be 0.87, 0.71, 0.83, 0.69, and 0.51, respectively. Overall alpha was also found to be 0.86 for this data. For CFA data obtained the reliability coefficients were found to be 0.89, 0.77, 0.71, 0.80, and 0.71, respectively. The overall alpha was also observed to be 0.85 for this data.
According to Pallant (2005), alpha values greater than 0.7 were acceptable for the reliability of the scale. The overall alphas were seen as acceptable values in this study so the questionnaire could be considered reliable to measure students' conceptions of the relationship between mathematics and physics.

Test-retest

While gathering the data to perform CFA, 30 students in the sample (N=338) were also identified to administer RMPQ at a different time as a retest. The students were contacted to re-administer the questionnaire after approximately one month. According to Pallant (2005), high correlations between the scores obtained from different occasions indicated a reliable scale. In this regard, correlations among students' scores on each factor obtained from different measurements were calculated. Test-retest reliability coefficients for factor 1, 2, 3, 4 and 5 were 0.87, 0.90, 0.91, 0.89 and 0.94, respectively. These values were also acceptable to claim that RMPQ was reliable.

Descriptive Results of RMPQ

Data used in EFA and CFA were combined to present the descriptive results. A total of 610 students' responses to RMPQ were examined. The distribution of the RMPQ responses to each item, in terms of frequencies and percentages, was as presented in Table 5.
Students' responses to each item in the RMPQ (see Appendix for actual item) revealed that the majority of students mostly agreed or strongly agreed to the statements of items. More than half of the students agreed or strongly agreed to the statements loaded on the first factor, which was 'content similarity conceptions'. They mostly viewed mathematics and physics as composed of rules, formulas, calculations, geometrical shapes, and numerical problems. They also believed that the two disciplines were largely based on memorization. Furthermore, according to most students, they were closely related to science and technology, and other disciplines.

Another conception students strictly held was about the similarities of mathematics and physics in terms of their relationship to daily-life. Most agreed or strongly agreed that these disciplines were necessary to overcome the problems, understand the nature, and improve living conditions in their life.

Table 5. Distribution of Students' Responses to RMPQ Items In Terms of Frequencies and Percentages

Factors	Items	Strongly disagree		Disagree		Neither agree nor disagree		Agree		Strongly agree	
		f	%	f	%	f	%	f	%	f	%
F1:	I17	63	10.3	39	6.4	69	11.3	153	25.1	286	46.9
CSC	I7	48	7.9	34	5.6	64	10.5	216	35.4	248	40.7
	I8	57	9.3	40	6.6	72	11.8	169	27.7	272	44.6
	I11	53	8.7	48	7.9	72	11.8	205	33.6	232	38.0
	I9	67	11.0	45	7.4	41	6.7	141	23.1	316	51.8
	I21	46	7.5	37	6.1	58	9.5	262	43.0	207	33.9
	I19	78	12.8	37	6.1	67	11.0	142	23.3	286	46.9
	I2	56	9.2	46	7.5	75	12.3	210	34.4	223	36.6
	I12	53	8.7	37	6.1	47	7.7	163	26.7	310	50.8
F2:	I1	112	18.4	97	15.9	113	18.5	151	24.8	137	22.5
MDC	I11	72	11.8	88	14.4	154	25.2	169	27.7	127	20.8
	I23	112	18.4	102	16.7	110	18.0	128	21.0	158	25.9
	I16	89	14.6	75	12.3	119	19.5	154	25.2	173	28.4
	I5	90	14.8	134	22.0	155	25.4	133	21.8	98	16.1
	I15	73	12.0	83	13.6	98	16.1	195	32.0	161	26.4
F3:	I22	80	13.1	88	14.4	163	26.7	146	23.9	133	21.8
CDC	I13	99	16.2	83	13.6	145	23.8	139	22.8	144	23.6
	I20	115	18.9	81	13.3	143	23.4	140	23.0	131	21.5
F4:	I4	94	15.4	65	10.7	126	20.7	156	25.6	169	27.7
DSC	I24	66	10.8	97	15.9	101	16.6	175	28.7	171	28.0
	I3	62	10.2	73	12.0	106	17.4	204	33.4	165	27.0
F5:	I10	55	9.0	71	11.6	168	27.5	159	26.1	157	25.7
DDC	I14	74	12.1	73	12.0	153	25.1	166	27.2	144	23.6
	I6	58	9.5	73	12.0	162	26.6	163	26.7	154	25.2

Almost half of the students agreed or strongly agreed that mathematics played a key role to succeed in physics. According to them, without having sufficient knowledge about mathematics, it was difficult to understand physics. They thought that mathematics was necessary to interpret the rules and formulas in physics, and solve physics problems. They viewed mathematics as a prerequisite to understand physics.

Additionally, students' conceptions about differences between mathematics and physics in terms of their content and frequency in daily-life use were tested due to RMPQ. While the majority of students believed that mathematics was more related to daily-life and most of the professions, they believed that physics was more related to natural phenomena. Moreover, in terms of content, physics was more complex than mathematics and included more rules than mathematics. They also pointed out that mathematics included more precise results than physics.

DISCUSSION

The aims of this study were as follows;

a) to determine high school students' conceptions of the relationship between mathematics and physics,
b) to develop a questionnaire measuring the conceptions.

The findings, gathered from the open-ended questionnaire and interview, revealed that some of the students' conceptions coincided with the findings and suppositions of the existing literature. For example, the abstract nature of mathematics (Cai, Perry, Wong & Wang, 2009) and physics (Ornek et al., 2007), use of rules and formulas (Korsunsky, 2002), the influence of understanding mathematics on successfulness in physics (Basson, 2002; Kapucu, 2014b; Semela, 2010) and the interaction of mathematics and physics with daily-life (Kapucu, 2014a), were widely discussed in the literature., The majority of students in this study agreed or strongly agreed with the items "successfulness in physics substantially depends on knowing mathematics" and "choosing a physics course depends on knowing mathematics". The amount of mathematics given in physics lessons might enable students to have such conceptions.

The above suggested science teachers, particularly physics teachers, should be more careful in teaching of physics topics. They should reduce the amount of mathematics given in science and physics courses. Instead, they should more emphasize what physics brings to our life to facilitate our living conditions and why physics was important for individuals. While teachers were exhibiting these behaviors, they should also give importance to conceptual understanding of physics concepts. As suggested in national high school physics curriculum (Ministry of National Education [MoNE], 2013), teachers should help students to discover the world around them by making connections among physics and their life experiences.

There was also little in the literature about the content differences and the daily-life differences between mathematics and physics. Although the previous studies indicated that both mathematics and physics included many rules and formulas to be remembered (Korsunsky, 2002) and both were difficult for students to understand (Basson, 2002), the students' conceptions showed a difference in the degree to which mathematics or physics included more rules and which one was more complex. Almost half of the students in this study agreed or strongly agreed with the items "physics includes more rules than mathematics to be memorized" and "physics is more complex than mathematics". Considering the neutral percentages about these items, it could be advocated that students view physics as more complex and memorization-based.

Students could have these conceptions because they might view mathematics as a discipline that included more related topics or concepts to each other. In other words, they could think that physics subjects were more independent from each other. Therefore, they could not easily construct the relationships among what they learn about physics when comparing to that about mathematics. Furthermore, as discussed by Clay et al. (2008), students should have difficulty in blending the knowledge of physics and mathematics. For example, they could have sufficient conceptual understanding of some physics concepts, but they could not solve some physics problems because they could not apply what they have learned in mathematics to the physics. Being not able to blend the knowledge of physics and mathematics in solving these physics problems could enable students to view physics as more complex and memorization-based.

In addition, almost half of the students agreed or strongly agreed that mathematics included more precise results than physics. We think that this result was not surprising, because students could generally focus on finding precise answer while solving mathematics problems. Especially in exams taken and in the school environment, mathematics teachers could expect final answers at the end of the students' problem solving processes. On the other hand, with the aim of explaining the nature, physics problems might seek for different answers under the specific scope of content, or topics.

Continuing with daily-life difference conceptions, more than half of the students agreed or strongly agreed with the items "mathematics is more related to most of the professions than physics" and "mathematics is more used in daily-life than physics". These results were indeed very interesting, according to us. We expected that students could not agree or strongly agree to these items too much, because of the nature of physics. As Basson (2002) indicated, physics is one of natural sciences that was based on observation of natural phenomena. Therefore, we expected that students should consider physics as more related to daily-life than mathematics. These conceptions might appear due to a lack of or inappropriate questions asked in classrooms and in textbooks (Korsunsky, 2002). Therefore, it should be necessary for physics teachers to expose students with more daily-life problems in classrooms.

Considering the factors obtained in RMPQ, other than math dependence, content and daily-life differences conceptions, there were two more factors, one of which was very considerable. Almost three-fourth of the students agreed or strongly agreed on each item in content similarity conceptions between mathematics and physics. Basically, students thought that both mathematics and physics required memorizations, the use of calculations, rules, formulas, geometrical shapes, and numerical problems. In general, all were widely discussed in the literature when explaining the interrelation between mathematics and physics (e.g., Ataide & Greca, 2013; Basson, 2002; Bing & Redish, 2006; Clay et al., 2008; Kapucu, 2014a; Kim & Aktan, 2014; Munier & Merle, 2009). Lastly, the daily-life similarity conception appeared as one important factor in RMPQ. Items in this factor was about improving our living conditions, overcoming problems in daily-life, and understanding the events in the universe. According to students' conceptions, both mathematics and physics helped us to do so. For example, physics qualitatively explained the physical phenomena in daily-life, while mathematics permitted quantitative predictions for it (Rohrlich, 2011). In addition, both were helpful for understanding the world (Kapucu, 2014a).

Based on the above, teachers should provide the logic behind the rules, formulas, and the concepts, which were considered as abstract, instead of just expecting students to memorize them. Students also focused on daily-life similarities and differences with respect to the relationship between mathematics and physics. It was possible that students had strong conceptions about daily-life relationship between mathematics and physics on learning them. Therefore, RMPQ might help teachers to be aware of students' conceptions about the relationship and then they should be more careful in how to help students to learn contents of mathematics and physics by considering the relationship.

Validity and reliability of RMPQ were also discussed considering its applicability in further studies. The data obtained led to the conclusion that RMPQ was satisfying, both in terms of validity and reliability.

CONCLUSION AND IMPLICATIONS

The relationship between mathematics and physics has an important place in literature and it is widely discussed by some researchers. Although some conceptions about this relationship are identified before, this study contributes to literature by presenting questionnaire (RMPQ) measuring high school students' conceptions of the relationship between mathematics and physics, RMPQ can be considered as an effective instrument to identify the conceptions.

As a recommendation for further studies,

(a) RMPQ could be subjected to larger samples. For example, a nation-wide study might be conducted to determine general conceptions among a nation's high school students about the relationship between mathematics and physics;
(b) the questionnaire could also be used to investigate, for example, the effectiveness of special instructional methods in interdisciplinary studies. This questionnaire gave information to the researchers about students' conceptions and what they were prone to be affected by in line with the relationship between mathematics and physics. So, it could be helpful for researchers to see the possible factors influencing the effectiveness of the methods used;
(c) researchers might use RMPQ to determine some reasons of students' failure in science courses. Students might have strong conceptions about mathematics dependence of physics so they could be unsuccessful in physics. Moreover, researchers should investigate the relationships among students' attitudes, beliefs or perceptions toward physics or its learning and their mathematics dependence conceptions to better understand the relationship between them. In this regard, the role of mathematics dependence conceptions in shaping students' psychological constructs (e.g., beliefs, attitudes, views) toward physics and its learning might be determined.

ACKNOWLEDGMENT

This study was funded by Ağrı İbrahim Çeçen University with a project number EF 14.009.

REFERENCES

Ataide, A., & Greca, I. (2013). Epistemic views of the relationship between physics and mathematics: Its influence on the approach of undergraduate students to problem solving. *Science & Education, 22*(6), 1405-1421.

Basson, I. (2002). Physics and mathematics as interrelated fields of thought development using acceleration as an example. *International Journal of Mathematical Education in Science and Technology, 33*(5), 679-690.

Bing, T. J., & Redish, E. F. (2006). The cognitive blending of mathematics and physics knowledge. *Proceedings of the Physics Education Research Conference,* Syracuse, NY.

Bogdan, R. C., & Biklen, S. K. (1998). *Qualitative research in education: An introduction to theory and methods (3rd ed.).* Needham Heights, MA: Allyn & Bacon.

Büyüköztürk, S. (2011). *Sosyalbilimleriçinverianalizi el kitabı*. Ankara: PegemAkademi.

Byrne, B. M. (2010). *Structural equation modeling with AMOS. Basic concepts, applications and programming.* Taylor & Francis Group: New York.

Cai, J., Perry, B., Wong, N. Y., & Wang, T. (2009). What is effective teaching? A study of experienced mathematics teachers from Australia, the Mainland China, Hong Kong – China, and the United States. In J. Cai, G. Kaiser, B. Perry, & N. Y. Wong (Eds.), *Effective mathematics teaching from teachers' perspectives: National and cross-national studies* (pp. 1- 36). Rotterdam, The Netherlands: Sense Publishers.

Clay, T. W., Fox, J. B., Grunbaum, D., & Jumars, P. A. (2008). How plankton swim: An interdisciplinary approach for using mathematics & physics to understand the biology of the natural world. *The American Biology Teacher, 70*(6), 363-370.

Costello, A. B., & Osborne, J. W. (2005). Best practices in exploratory factor analysis: Four recommendations for getting the most from your analysis. *Practical Assessment, Research & Evaluation, 10*(7), 1-9.

Eklund-Myrskog, G. (1998). Students' conceptions of learning in different educational contexts. *Higher Education, 35*(3), 299-316.

Entwistle, N. (1997). Introduction: Phenomenography in higher education. *Higher Education Research & Development, 16*(2), 127-134.

Fauconnier, G., & Turner, M. (2002). *The way we think: Conceptual blending and the mind's hidden complexities*. New York: Basic Books

Heller, M. (1997). Essential tension: Mathematics-physics-philosophy. *Foundations of Science, 2*(1), 39-52.

Hermann, S. (1991). Problem solving in physics and mathematics – What do they have in common? (Creating a base for research and teaching). Paper presented at the *Annual Meeting of the National Association for Research in Science Teaching*, Lake Geneva, WI.

Hudson, H. T., & McIntire, W. R. (1977). Correlation between mathematical skills and success in physics. *American Journal of Physics, 45*(5), 470–471.

Jitendra, A. K., Griffin, C. C., & Xin, Y. P. (2010). An evaluation of the intended and implemented curricula's adherence to the NCTM Standards on the mathematics achievement of third grade students: A case study. *Journal of Curriculum Instruction, 4*(2), 33-50.

Kapucu, S. (2014a). University students' conceptions of the relationship between mathematics and physics and the relationship between mathematics and physics learning. *Journal of Baltic Science Education*, *13*(5), 622-636.

Kapucu, S. (2014b). Salient beliefs of pre-service primary school teachers underlying an attitude "liking or disliking physics". *Science Education International, 25*(4). 437-458.

Kim, M., & Aktan, T. (2014). How to enlarge the scope of the curriculum integration of mathematics and science (CIMAS): A delphi study. *Eurasia Journal of Mathematics, Science, and Technology Education, 10*(5), 455-469.

Korsunsky, B. (2002). Improper use of physics-related context in high school mathematics problems: Implications for learning and teaching. *School Science and Mathematics, 102*(3), 107-113.

Linder, C., & Marshall, D. (2003). Reflection and phenomenography: Towards theoretical and educational development possibilities. *Learning and Instruction, 13*(3), 271-284.

Martinez-Torregrosa, J., Lopez-Gay, R., & Gras-Marti, A. (2006). Mathematics in physics education: Scanning historical evolution of the differential to find a more appropriate model for teaching differential calculus in physics. *Science & Education, 15*(5), 447-462.

Mestre, J. P. (2001). Implications of research on learning for the education of prospective science and physics teachers. *Physics Education, 36*(1), 44-51.

Ministry of National Education (MoNE) (2013). *National high school physics curriculum.* Ankara: Talim ve Terbiye Kurulu Başkanlığı

Munier, V., & Merle, H. (2009). Interdisciplinary mathematics–physics approaches to teaching the concept of angle in elementary school. *International Journal of Science Education, 31*(14), 1857-1895.

Nalçacı, İ. Ö., Akarsu, B., & Kariper, İ. A. (2011). Ortaöğretim öğrencileri için fizik tutum ölçeği derlemesi ve öğrenci tutumlarının değerlendirilmesi. *Journal of European Education, 1*(1), 1-6.

National Council of Teachers of Mathematics (NCTM) (2000). *Principles and standards for school mathematics*. Reston, VA.

Olatoye, R. A. (2007). Effect of further mathematics on students' achievement in mathematics, biology, chemistry and physics. *International Journal of Environmental & Science Education, 2*(2), 48-53.

Ornek, F. (2008). An overview of a theoretical framework of phenomenography in qualitative education research: An example from physics education research. *Asia-Pacific Forum on Science Learning and Teaching, 9*(2).

Ornek, F., Robinson, W. R., &Haugan, M. R. (2007). What makes physics difficult? *Science Education International, 18*(3), 165-172.

Pallant, J. (2005). *SPSS survival manual: A step by step guide to data analysis using SPSS for Windows (Version 12)*. Crows Nest NSW, Australia: Allen & Unwin.

Rohrlich, F. (2011). Why physics needs mathematics. *General Relativity and Gravitation, 43*(12), 3665-3669.

Scherer, R. F., Wiebe, F. A., Luther, D. C., & Adams J. S. (1988). Dimensionality of coping: Factor stability using the ways of coping questionnaire. *Psychological Reports, 62*(3), 763-770.

Schermelleh-Engel, K., & Moosbrugger, H. (2003). Evaluating the fit of structural equation models: Tests of significance and descriptive goodness-of-fit measures. *Methods of Psychological Research Online, 8*(2), 23-74.

Semela, T. (2010). Who is joining physics and why? Factors influencing the choice of physics among Ethiopian university students? *International Journal of Environmental & Science Education, 5*(3), 319-340.

Toluk Uçar, Z., Pişkin, M., Akkaş, E. N., & Taşçı, D. (2010). Elementary students' beliefs about mathematics, mathematics teachers, and mathematicians. *Education and Science, 35*(155), 131-144.

Tsai, C. C. (2004). Conceptions of learning science among high school students in Taiwan: A phenomenographic analysis. *International Journal of Science Education, 26*(14), 1733-1750.

Woolnough, J. (2000). How do students learn to apply their mathematical knowledge to interpret graphs in physics? *Research in Science Education, 30*(3), 259-267.

Worthington, R. L., & Whittaker, T. A. (2006). Scale development research: A content analysis and recommendations for best practices. *The Counseling Psychologist, 34*(6), 806-838.

Appendix

Items included in the RMPQ

I17	Mathematics and physics are mostly based on memorization.
I7	Mathematics and physics mostly include calculations.
I8	Mathematics and physics are closely related to science and technology.
I11	Mathematics and physics mostly use rules or formulas.
I9	Mathematics and physics mostly include numerical problems.
I21	Mathematics and physics use geometrical shapes, tables and diagrams.
I19	Mathematics and physics are related to many disciplines.
I2	Mathematics and physics are composed of related topics with each other.
I12	Mathematics and physics include abstract concepts.
I1	Physics topics are not deeply understood without knowing mathematics.
I11	Physics problems are not solved without knowing mathematics.
I23	Physics rules and formulas are not interpreted without knowing mathematics.
I16	Successfulness in physics substantially depends on knowing mathematics.
I5	Choosing a physics course depends on knowing mathematics.
I15	Physics phenomena are not explained without knowing mathematics.
I22	Physics includes more rules than mathematics to be memorized.
I13	Physics is more complex than mathematics.
I20	Mathematics includes more precise results than physics.
I4	Mathematics and physics help us to improve our living conditions.
I24	Mathematics and physics play an important role to overcome the problems in our life.
I3	Mathematics and physics help us to understand the events in the universe.
I10	Mathematics is more related to most of the professions than physics.
I14	Mathematics is more used in daily life than physics.
I6	Physics is more related to natural phenomena than mathematics.